Estimation of an individual's Human Cone Fundamentals from their Color Matching Functions

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Abstract

Estimating individual cone sensitivities on the basis of their corresponding individual color matching functions is a classic problem of color vision. To solve the problem one has, in effect, to postulate constraints on the shape of the cone sensitivities. For example, Logvinenko applied a linear matching method (originally proposed by Bongard and Smirnov [4]) which excludes all but one primary to which each of the sensitivities of the visual system is supposedly sensitive and, mathematically, if this constraint is met reasonable estimates should result. However, when applied to color matching functions in general, it turns out that the method estimates middle and especially long wave sensitivity quite poorly. We propose a new method based on linear optimization, in which it is assumed only that the photo-pigment spectral absorptance functions are known a priori. In an iterative scheme, the method works by simultaneously estimating the coefficients of the linear relation between the known individual color matching functions and estimated cone sensitivities and estimating the ocular and macular filtering that multiplied by the absorptances yield the estimated cone sensitivities. The ocular and macular pre-filtration is treated as a single spectral function (i.e. a combined ocular and macular filtration). We are also able to predict the cone sensitivities proposed by Stockman and Sharpe in recent research. The method is tested on a selection of individual 1959 10 degree Color Matching Functions, with the assumption that they have the Stockman and Sharpe 10 degree photo-pigment spectral absorbtance in common. The results for the estimated cone sensitivities look very plausible. Finally we have applied the method to the CIE1964 10 degree observer and get very reasonable result as well.

Introduction

The characterization of the human color vision [6], [7], [8] has been accomplished through the development of a standardized human observer. This standard observer has been adopted by industrial and scientific communities, even though individual differences are known to exist between observers. The most recently recommended modified standard observer (2 and 10 degree viewing angles) are presented in the CIE technical report (TC 1-36) [5], a work comprehensively based on the work of Stockman and Sharpe [6]. These sensitivities are based on averaged trichromatic psychophysical data and data from dichromats, in conjunction with the König assumption that dichromats are a reduced form of trichromats, by the simple lack of either one of the cone sensitivities. These estimations are thus inherently not individual. In general the human vision differs from that standard [2] [1] due to deficiency and anomaly. In particular, for the normal trichromatic vision, the differences are due to polymorphism and optical density variations in the ocular media, macular pigmentation and photo-pigmentation. Rather than attempting to model the individual human vision, the aim is to apply mathematical methods, incorporating a minimum of physically feasible constraints, to existing and well-established spectrally characterizing data for the human vision, in the shape of color matching functions, in order to estimate individual cone fundamentals. A classical linear method (originally proposed by Bongard and Smirnov [4]) used by Lobanova and Speranskaya [10] [11] in order to study anomaly and revisited by Logvinenko [3] with the purpose of estimating individual cone sensitivities from color matching functions forms an inspirational background. Problems with especially the estimation of the long-wave cone fundamental are consistently reported.

In an attempt to enhance the estimation of individual cone fundamentals based on color matching functions a new approach to solve the problem is presented. The method most importantly relies on the constraint, that for a given set of color matching functions, the corresponding photo-pigment absorptances (effective photo-pigment sensitivities) are known.

In order to know these the photo-pigment spectral absorbance functions (i.e. recently estimated and tabulated in [6] and found on the basis of an average of the Stiles-Burch 10 degree 1959 color matching functions), their peak wavelengths (polymorphic version) and their peak optical densities (thickness of photo-pigment layer) must be known. Assuming these are indeed correct for the set of color matching functions, the method will estimate a pre-filter, which constitutes a combined macular and ocular filtration function, along with the linear relation between the color matching functions and the resulting estimated cone fundamentals. The method also includes the physically feasible constraint, that the estimated cone fundamentals must be non-negative.

Thus, when applying these constraints to a broader set of individual color matching functions, it is indirectly assumed, that the main source of variation between individuals stems from differences in the absorption in the ocular media and the macular pigmentation: A sizable contributor to this variation is age related pigmentation changes in the lens. Mathematically combining the two absorption types into one pre-filter means that the pre-filter constitutes the difference between corneal and effective photo-pigment sensitivity. The pre-filter covers all spectral filtration in the light path from cornea to the outer segment of the cones.

As an approximation to those, the photo-pigment absorptances used in the Stockman-Sharpe 10 degree cone fundamentals are used in the experiments.

Problem statement

An observer's spectral cone sensitivities $\mathbf{LMS}(\lambda)$ and color matching functions $\mathbf{CMF}(\lambda)$ are as mentioned before linearly related.

$$\mathbf{LMS}(\lambda) = \mathbf{CMF}(\lambda) \cdot \mathbf{M} \tag{1}$$

Where:

$$\mathbf{LMS}(\lambda) = [\bar{l}(\lambda)\ \overline{m}(\lambda)\ \bar{s}(\lambda)] \tag{2}$$

and

$$\mathbf{CMF}(\lambda) = [\bar{r}(\lambda)\ \bar{g}(\lambda)\ \bar{b}(\lambda)] \tag{3}$$

and:

$$\mathbf{M} = \begin{bmatrix} l(e_1) & m(e_1) & s(e_1) \\ l(e_2) & m(e_2) & s(e_2) \\ l(e_3) & m(e_3) & s(e_3) \end{bmatrix} \tag{4}$$

where e_1, e_2, e_3 are the primaries corresponding to the color matching functions in $\mathbf{CMF}(\lambda)$. For any individual set of color matching functions, the problem is to find the 9 coefficients $M_{i,j}$ in $\mathbf{M}$ that transform the color matching functions into the corresponding spectral cone sensitivities.

The cone fundamentals are related to the retinal photo-pigment spectral absorptance $\mathbf{PDT}(\lambda)$ by the lens- and macular optical density functions combined in one pre-filter function $F(\lambda)$ [5]. In continuous form:

$$\mathbf{LMS}(\lambda) = F(\lambda)\mathbf{PDT}(\lambda) \tag{5}$$

where:

$$\mathbf{PDT}(\lambda) = [\bar{p}(\lambda)\ \bar{d}(\lambda)\ \bar{t}(\lambda)] \tag{6}$$

and $\bar{p}(\lambda)$ is the long wave, $\bar{d}(\lambda)$ is the middle wave and $\bar{t}(\lambda)$ short wave photo-pigment spectral absorptance. The functions are shown in Figure 2.

In general spectral absorptance describes how much light is actually absorbed by a pigment given the optical density of the pigment layer [5]. Given the low density spectral absorbance and the correponding peak optical density, the spectral absorptance can be calculated: The spectral absorbtance expressed in terms of energy units for the long wave photo-pigment:

$$\bar{p}(\lambda) = k^p(1 - 10^{-D^p A^p_\lambda})\lambda \tag{7}$$

the middle wave photo-pigment:

$$\bar{p}(\lambda) = k^d(1 - 10^{-D^d A^d_\lambda})\lambda \tag{8}$$

and the short wave photo-pigment:

$$\bar{p}(\lambda) = k^t(1 - 10^{-D^t A^t_\lambda})\lambda \tag{9}$$

where k^p, k^d and k^t are factors for normalizing to unit peak. The peak photo-pigment optical densities for long wave D^p, middle wave D^d and short wave D^t are a scalars. With regard to the 10 degree Stockman and Sharpe cone fundamentals, these densities are shown in the bottom row of Table 1 . The corresponding low density spectral absorbances A^p_λ, A^d_λ and A^t_λ for each of the three photo-pigments are tabulated in [5] (in normalized logarithmic quantal units on wavelength basis) and shown in Figure 1.

Peak Photo-pigment Optical Densities			
Observer	D^p	D^d	D^t
10 deg.	0.38	0.38	0.30

Table 1: The optical densities of the Stockman and Sharpe photo-pigments for the 10 degree viewing field.

The optical density varies over the surface of the retina as the thickness of the pigment layer varies. Toward the center of the retina (foveola) the pigment layer thickens as the concentration of cones increases and thus the optical density becomes larger. This means that there is an important difference between the absorptance spectra of foveal and parafoveal vision. This difference is one of the reasons for the partition of standard observers into small field 2 degree and large field 10 degree observers. The absorptance spectra broadens (the peak stays the same) as D increases. Inserting Equation 5 into Equation 1 yields:

$$F(\lambda)\mathbf{PDT}(\lambda) = \mathbf{CMF}(\lambda) \cdot \mathbf{M} \tag{10}$$

The objective is to estimate the individual pre-filter $F(\lambda)$ and the individual matrix $\mathbf{M}$, based on the assumption that the photo-pigment absorptances $\mathbf{PDT}(\lambda)$ is known and in common for each of the individual color matching functions $\mathbf{CMF}(\lambda)$.

Method

The individual color matching functions $\mathbf{CMF}(\lambda)$ are known and we assume photo pigments $\mathbf{PDT}(\lambda)$ are also known

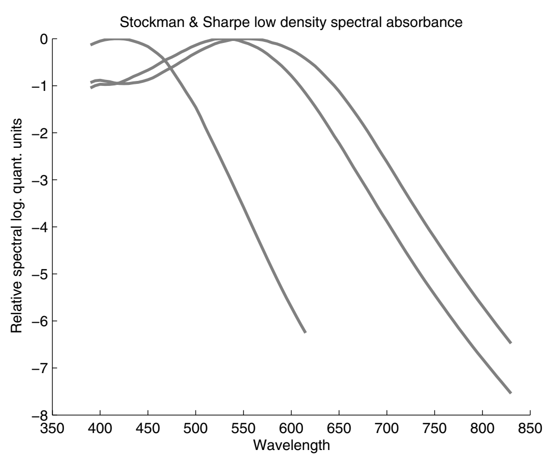

Figure 1. *Plot of the Stockman Sharpe low density spectral absorbances A^p_λ, A^d_λ and A^t_λ for each of the three photo-pigments.*

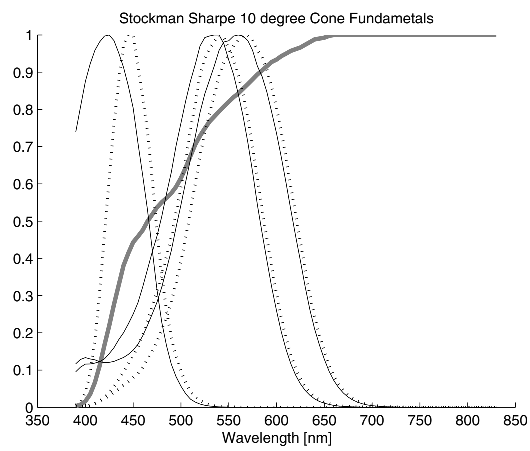

Figure 2. *Plot of the Stockman Sharpe cone fundamentals (3 dotted curves) and the corresponding photo-pigment absorptances (3 full gray curves) and the combined ocular and macular pre-filter (full thick curve) as a function of wavelength. The functions are all normalized to unit peak value*

(i.e. same for different observers). Unknown is the individual pre-filter $F(\lambda)$ and the 3 by 3 matrix **M** that relates color matching functions to estimated cone functions as stated in Equation 1. Rearranging Equation 10 by division of $F(\lambda)$ the problem becomes:

$$\mathbf{PDT}(\lambda) = F^{-1}(\lambda) \cdot \mathbf{CMF}(\lambda) \cdot \mathbf{M} \tag{11}$$

to be solved for **M** and $F(\lambda)$. In discreet formulation Equation 11 becomes:

$$\mathbf{PDT} = \mathbf{D} \cdot \mathbf{CMF} \cdot \mathbf{M} \tag{12}$$

D is $N \times N$ diagonal matrix with the inverse filtering function $F^{-1}(\lambda)$ in discreet representation, along the diagonal. **CMF** is $N \times 3$ and **M** is 3×3. Multiplying these together matrix-wise returns the N x 3 **PDT** matrix.

Solving for **D** and **M** is carried out by rearranging Equation 12 to define a problem of minimization:

$$I = ||\mathbf{D} \cdot \mathbf{CMF} \cdot \mathbf{M} - \mathbf{PDT}||^2 \tag{13}$$

in an iterative scheme of two sets of equations. Firstly there is a set of N equations of minimization:

$$I_i^{k+1} = ||\mathbf{D}_{ii}^{k+1} \cdot (\mathbf{CMF}^k \cdot \mathbf{M}^k)_i - \mathbf{PDT}_i||^2 \tag{14}$$

where $i = (1..N)$ to solve for the pre-filter consisting of diagonal elements $\mathbf{D}_{ii}$.

Secondly a set of 3 equations:

$$I_j^{k+1} = ||(\mathbf{D}^{k+1} \cdot \mathbf{CMF}^k) \cdot \mathbf{M}_j^{k+1} - \mathbf{PDT}_j||^2 \tag{15}$$

to solve for **M**. In order to incorporate the physically feasible constraint of non-negativity of the estimated cone fundamentals, Equation 15 is combined with:

$$(\mathbf{D}^{k+1} \cdot \mathbf{CMF}^k) \cdot \mathbf{M}_j^{k+1} \geq 0 \tag{16}$$

for $j = 1,2,3$.

The solution is found by iterating alternately between Equation 14 and Equations 15 and 16.

The solution to equation 14 is found by using the Moore-Penrose pseudo-inverse to obtain the diagonal coefficients $\mathbf{D}_{ii}$ for $i = 1..N$:

$$\mathbf{D}_{ii}^{k+1} = \mathbf{PDT}_i \cdot (\mathbf{CMF}^k \cdot \mathbf{M}^k)_i^{-1} \tag{17}$$

To solve for **M** by Equations 15 and 16 then Equation 13 is expanded to:

$$I_j = (\mathbf{D} \cdot \mathbf{CMF}^k \cdot \mathbf{M}_j - \mathbf{PDT}_j)^T (\mathbf{D} \cdot \mathbf{CMF}^k \cdot \mathbf{M}_j - \mathbf{PDT}_j) \tag{18}$$

Differentiating Equation 18 with respect to the three variables $\mathbf{M}_j$ and setting the resulting equations equal to zero yields:

$$\partial I_j / \partial \mathbf{M}_j = 0 \tag{19}$$

so that:

$$(\mathbf{D} \cdot \mathbf{CMF}^k)^T \cdot (\mathbf{D} \cdot \mathbf{CMF}^k) \cdot \mathbf{M}_j - \cdot \mathbf{PDT}_j^T \cdot (\mathbf{D} \cdot \mathbf{CMF}^k) = 0 \tag{20}$$

Equation 20 is a quadratic programming problem to be solved for the variables $\mathbf{M}_j$ with the non-negativity constraint in Equation 16. The variables **D** is and **M** are at the iteration level as indicated in Equation 15. In order to link the solutions from the two sets of Equations 17 and 20 $\mathbf{CMF}^k$ is updated by:

$$\mathbf{CMF}^{k+1} = \mathbf{D}^{k+1/2} \cdot \mathbf{CMF}^k \cdot \mathbf{M}^{k+1} \tag{21}$$

Iterations

Initializing the iterative solution in k^{stop} iterations each of which consisting of five steps, with $k = 0$:

$$\mathbf{CMF}^0 = \mathbf{CMF} \tag{22}$$

$$\mathbf{D}^0 = diag(1, 1...1) \tag{23}$$

$$\mathbf{M}^0 = diag(1, 1, 1) \tag{24}$$

$$\mathbf{M}_r = \mathbf{M}^0 \tag{25}$$

$$\mathbf{D}_r = \mathbf{D}^0 \tag{26}$$

The first step is to solve for **D** by Equation 17. The second step is to solve for **M** by the quadratic programming solution defined by Equations 20 and 16. The color matching functions $\mathbf{CMF}^k$ are updated by Equation 21 in a third step. The fourth step is to memorize **M** and **D** by:

$$\mathbf{M}_r = \mathbf{M}_r \cdot \mathbf{M}^{k+1} \tag{27}$$

and

$$\mathbf{D}_r = \mathbf{D}_r \cdot \mathbf{D}^{k+1} \tag{28}$$

The fifth step is to update k to $k+1$. The iterations run through these steps until a stop criteria is met. The stop criteria is defined as the iteration number k^{stop} where $I^{k_{stop}+1} - I^{k_{stop}} < \varepsilon$ where $\varepsilon = 0.001$:

$$I^{k_{stop}} = ||\mathbf{D}_r \cdot \mathbf{CMF} \cdot \mathbf{M}_r - \mathbf{PDT}||^2 \tag{29}$$

The final solutions for **M** and **D** are calculated by:

$$\mathbf{M}_r = \prod_{k=0}^{k_{stop}} \mathbf{M}^k \tag{30}$$

and

$$\mathbf{D}_r = \prod_{k=k_{stop}}^{0} \mathbf{D}^k \tag{31}$$

which means that the estimated cone fundamentals are:

$$\mathbf{LMS} = \mathbf{CMF} \cdot \mathbf{M}_r \tag{32}$$

and the estimated pre-filter is diagonal elements of $\mathbf{D}_r^{-1}$.

Results

We have applied the the method to the Stiles-Burch 10 degree 1959 slit width corrected color matching function data. Of the 53 data sets of color matching functions 51 was chosen: Two sets suffers from lack of complete data. The data is given on wavenumber basis and thus needs re-interpolation on wavelength basis which was carried out using a standard polynomial piecewise third order spline function. The relation between λ (wavelength in [nm]) and wavenumber wn is:

$$\lambda = 10^7/wn \tag{33}$$

The wavelength intervals chosen, between 395 and 710 nm, was 5 nm which approximately is the interval which polymorphic variation (variation in absorptances peak wavelength value) covers. The constraining absorptance functions were the Stockman and Sharpe 10 degree data (converted from relative quantal absorbance spectra in log units to wavelength based energy units). This was carried out by using Equations 7, 8 and 9 with the optical peak densities given in Table 1 .

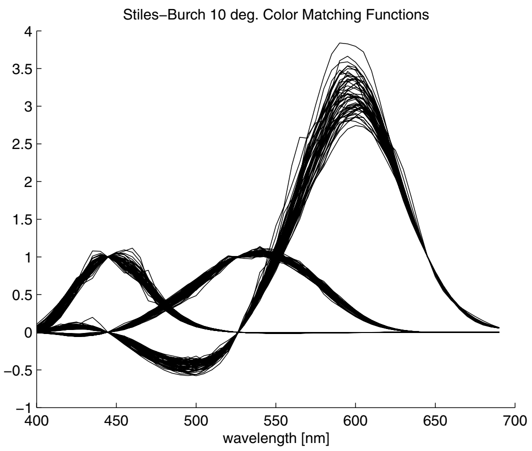

Figure 3. Plot on wavelength basis of the Stiles-Burch 10 degree 1959 color matching functions from which 51 have been chosen. The functions are all normalized to unit at the locations of the primaries

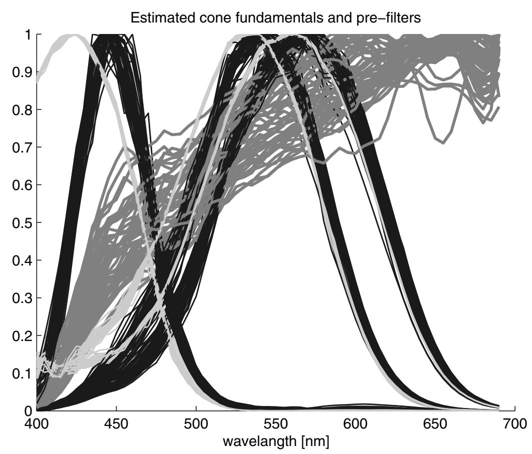

Figure 4. Plot on wavelength basis of the estimated cone fundamentals (black curves), the estimated pre-filters (dark gray curves) and the individual photo-pigment absorptances resulting from dividing cone fundamentals with the corresponding pre-filter (light gray curves). The functions are all normalized to unit at peak wavelength

The results are shown in the figures. In Figure 2 the Stockman and Sharpe 10-degree cone fundamentals and their corresponding absorptances and pre-filter is shown. In Figure 3 we plot the 51 chosen individual Stiles-Burch 10-degree color matching functions, that are subjected to our method. The results of applying our method are shown in Figure 4. The thick dark gray curves are the estimations of each pre-filter pertaining to each cone fundamental estimation shown in black curves. The light gray curves are the results of dividing the cone fundamentals by the pre-filters; these curves give an indication of how the error in the estimation relates to the constraint defined by the absorptance functions. To elucidate these results further we plot them in Figure 5 for a single observer (the data belongs to Arthur Tarrant aged 27). The similarities between our outputs and Stockman and Sharpe functions are self-evident, although we would always expect some difference, since our functions are derived for a single observer and not the standard observer. We have also applied the method to the CIE1964 10-degree standard observer and the result is shown in Figure 6. Having said this, the normalization of the pre-filter to unit peak value is somewhat arbitrary and the jittery behaviour of the functions beyond 600 nm has a pronounced influence on the visualization in the graph.

Conclusion and discussion

In this paper we have presented a new method to estimate individual cone fundamentals based on the corresponding color matching functions. The method additionally estimates

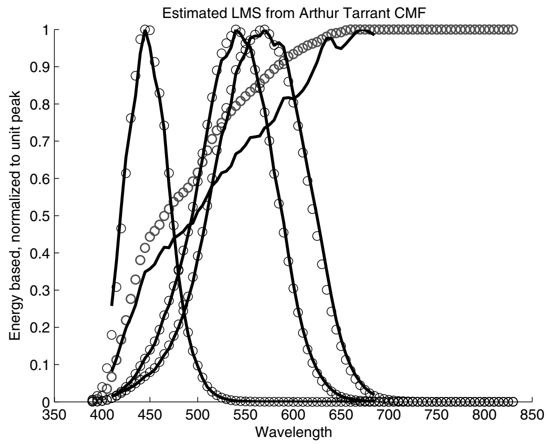

Figure 5. Plot of the Stockman Sharpe cone fundamentals and pre-filter (circles) shown in Figure 2 superimposed with the estimations of cone fundamentals and pre-filter (thick full black curves) for Arthur Tarrant who was among the persons participating in the 1959 color matching experiment. The functions are all normalized to unit peak value

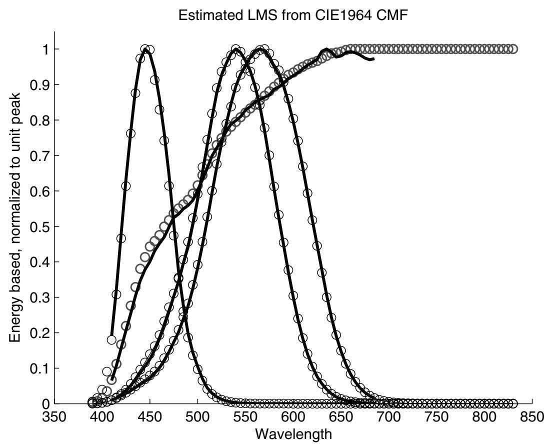

Figure 6. Plot of the Stockman Sharpe cone fundamentals and pre-filter (circles) shown in Figure 2 superimposed with the estimations of cone fundamentals and pre-filter (thick full black curves) for the CIE 1964 10 degree standard observer color matching functions. The functions are all normalized to unit peak value

a pre-filter that constitutes the combined macular and ocular pre-filtration. The method is iterative and based on linear optimization and it relies on the assumption that the photo-pigment spectral absorptances pertaining to the individual color matching functions are known a priori. We have applyed the method on the set of individual Stiles-Burch 10 degree color matching functions with the assumption that they have the Stockman and Sharpe photo-pigment spectral aborptances in common. The results show that the proposed method gives cone fundamentals that are remarkably similar to those derived by Stockman and Sharpe from the Stiles-Burch 10-degree color matching functions, even though we use a more general constraint, that does not take into account variation in peak optical densities and peak wavelengths. Thus, we expect that some variation between the true individual cone fundamentals and the estimated is present in the results. While we have no objective measure for the quality of these estimates, the performance of the method on the Stiles-Burch 10-degree color matching functions suggests that they will be close to the true fundamentals. We propose to apply the method to the Stiles-Burch 2 degree pilot data as well. In the future we propose to incorporate allowance for variations in peak wavelength and peak optical densities in the method as well. It will also be possible to reduce the amount of color matching function samplings significantly as the method is basically solving for only up to nine coefficients in the linear combination that relates color matching functions to estimated cone fundamentals.

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