A Model for Jet Shortening in Drop-On-Demand Ink-Jet Printing

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Abstract

A new model has been developed for the surface energy-driven shortening of a free, cone-shaped fluid ligament of finite length, as a function of ligament diameter, length, mass and head speed. It differs significantly from classical models based on infinitely long cylindrical (Taylor) or conical (Keller) shapes, but leads to overall shortening speeds which are very similar to those provided by Taylor's model for typical drop-on-demand fluids.

However, if a realistic initial velocity distribution along the length of the ligament is included, the model predicts more rapid shortening, by as much as 2 m/s for a jet speed of 6 m/s. Such effects should be taken into account when analyzing the behavior of real jets.

The model's predictions of shortening speeds for free drop-on-demand jets fail to account for all experimental observations, which for some polymer solutions can be as much as 2-3 times as high. This effect is attributed to elastic retraction, and may be a general feature linked to the polymer relaxation time.

Previous models

Higher drop-on-demand (DoD) ink-jet speeds generally involve longer fluid ligaments and later detachment of the jet from the nozzle. Reliable estimates of the time required to form a final drop would be valuable in the design of high speed printing or deposition processes, in setting-up or specifying parameters such as drop velocity, substrate position and printing rate, for almost all types of ink-jet fluid. We focus here on the speed with which the fluid ligament length shortens.

Theoretical treatments of free ligament shortening after break-off based on static cylinders or cones have continued to be used, despite the availability of precise measurements of the shapes and speeds of extending ink-jets [1-3]. Reasons for this have included their simplicity and the apparent success of the classical results for the shortening speed, derived from Taylor's [4] model for the case of the bounding rim formed on a thin fluid sheet formed by impacting jets. Another reason is the attention paid to fluids with low viscosity, where the ligament tends to pinch off at both ends, leaving a thin fluid body that can reasonably be represented by a cylindrical shape during the early recoil time. However, ligament shapes measured for more viscous fluids are approximately conical and comprise fluid with an internal, axial velocity distribution prior to break-off.

Keller [5] surmised that, for the breaking of threads (and films) of various geometries, some useful relations could be deduced on the basis of a power law in the radial (relevant) dimension. Power laws assume the same behavior right down to vanishing radius, so that after break-off the broken tip has zero mass and infinite recoil speed, for a conically shaped ligament. The subsequent tail shortening speed always decreases with increasing time in this model and therefore never reaches a constant value.

Such results have been recently used without comment [1], although the dynamics of pinching and breaking of viscous threads involve some very complex phenomena [6]. In this work we explore how finite jet length and more realistic assumptions of jet shape and internal velocity distribution can be incorporated into a model for ligament shortening, and compare its predictions with those from the Keller and Taylor models. Shortening speeds are then compared with those measured in jetting experiments with Newtonian and viscoelastic fluids.

Jetting fluid is often characterized by its density ρ, surface tension σ and viscosity η, but influences of viscosity or elasticity are not included in any of the models. The shortening of the fluid ligament is driven by surface tension and mass is conserved.

Figure 1 shows the geometries assumed in (a) the classical model for a cylindrical ligament (Taylor), and (b) the model for a conical ligament (Keller) [5]. In the Taylor model the diameter D is constant but for cones it increases linearly with x.

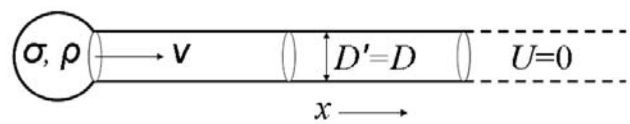

Figure 1(a): The basis of the classical (Taylor) model for shortening of a cylindrical ligament. The ligament is assumed to be infinitely long and stationary (U=0), with no internal velocity distribution. The end mass m grows linearly with time, and the ligament exhibits a constant shortening speed given by $v_T = 2(\sigma/\rho D)^{1/2}$.

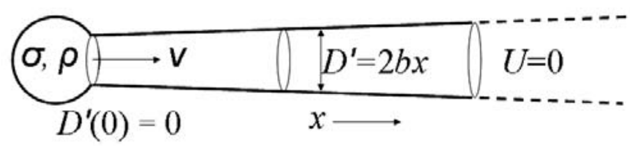

Figure 1(b): The basis of the (Keller) model for shortening of a conical ligament (where D' = 2bx). The shortening speed is given by $v(t) = (8\sigma/5\rho bt)^{1/3}$ and thus varies with time. The mean speed over time T is <v> = (3/2) v(T).

Proposed model

Figure 2 shows the model proposed for a truncated conical ligament with initial length L, including an initial axial speed distribution which varies linearly along the ligament. The total ligament mass M and the tail end mass m experience equal and opposite surface forces at the ligament diameter D', and the remaining part has mass (M-m), here shown lumped into the head. Both ends are assumed to remain attached to the ligament throughout shortening; the viscosity and elasticity are also ignored.

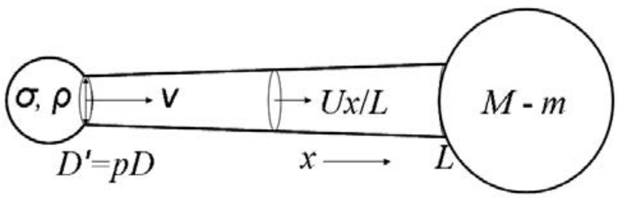

Figure 2: The basis of the present model for shortening of a truncated conical ligament with initial length L. The total (conserved) mass is M and there is an initial axial velocity profile due to ligament extension between the break-off position (close to the nozzle) and the tip. Further details are described in [7].

The acceleration for the tail (and head) can be expressed in terms of the physical and the geometrical parameters of the problem, and a full derivation is given elsewhere [7]. The classical Taylor model for an infinite cylindrical ligament (Fig. 1a) predicts a constant jet shortening speed v_T given by:

$$v_T = 2\sqrt{\frac{\sigma}{\rho D}} \tag{1}$$

At the break-off point, if the cylindrical ligament is assumed to have a hemispherical end-cap of diameter D at break-off, the acceleration of this free end mass, for a cylindrical ligament, after time t where $t = 0$ at break-off, is given by

$$\frac{dv}{dt} = 3\frac{v_T^2 - v^2}{D + 3x} \tag{2}$$

where x is the reduction in the length L of the original ligament in the time t. The assumption in the Taylor model is that the cylinder is stationary: equation (2) shows that the tail shortens at a steady rate given by equation (1), whether the (unknown) initial tail shortening velocity v (t=0) at break-off is greater or smaller than the steady-state value v_T. The assumption of the initial hemispherical end introduces the diameter D into the denominator of equation (2). This choice is important only if x is small, and other representations of the initial end shape can be explored through equation (2) at constant v_T; for example, if D=0, the equilibrium speed v_T is attained instantaneously after break-off.

When the ligament is not cylindrical, but a truncated cone of length L, and with an initial hemispherical end, the effects of the ligament shape enter through the instantaneous value of $p=D'/D$, while the introduction of an initial axial speed distribution which varies linearly along the ligament (at break-off) introduces two terms involving (U/L), to give equation (3):

$$\frac{dv}{dt} = 3\frac{v_T^2 p - vp^2(v - xU/L)/(1 + Ut/L)}{D + x(1 + p + p^2)} \tag{3}$$

Equation (3) shows that for cylinders (where p=1), the initial linear axial velocity distribution for $U > 0$ always increases the tail acceleration, whereas without an initial axial gradient (where U=0) conical shapes with $p > 1$ always reduce the tail acceleration; with both U=0 and p=1, equation (3) reduces to equation (2).

The head mass (M-m) acceleration can be written in terms of the shortening speed from the head end V and mass m:

$$\frac{dV}{dt} = -\frac{1}{(M - m)}(\pi D\sigma - V\frac{dm}{dt}) \tag{4}$$

The time variation of tail mass m is obtained as part of the full derivation of equation (3) and the total ligament shortening speed can be determined by integrations of equations (3) & (4) [7].

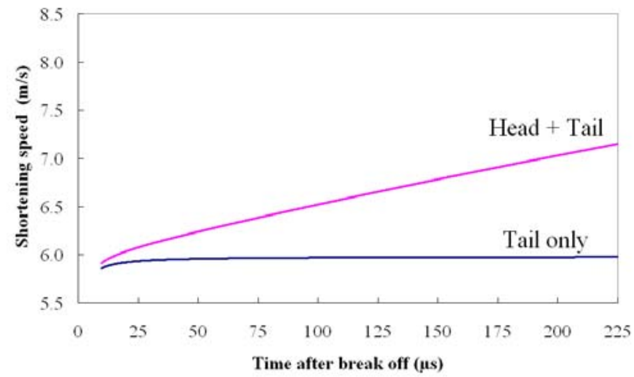

Figure 3(a): The shortening speed predicted for a finite cylindrical ligament with a typical diameter of 3.7 µm [2]. The 'tail only' curve shows a constant shortening speed (i.e. the Taylor prediction), while the upper curve which incorporates both head and tail motion shows a steadily increasing speed.

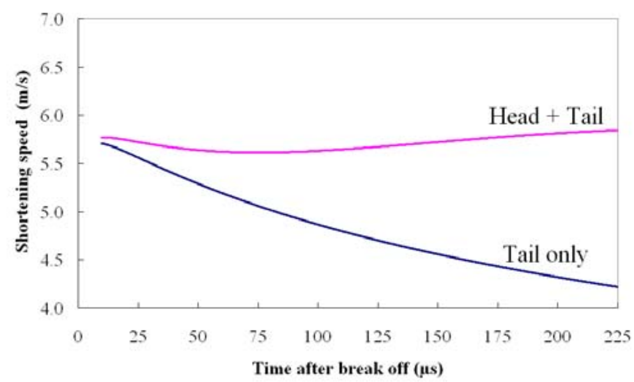

Figure 3(b): The shortening speeds predicted for a long, truncated, conical ligament. There is no initial axial velocity profile within the ligament (U=0).

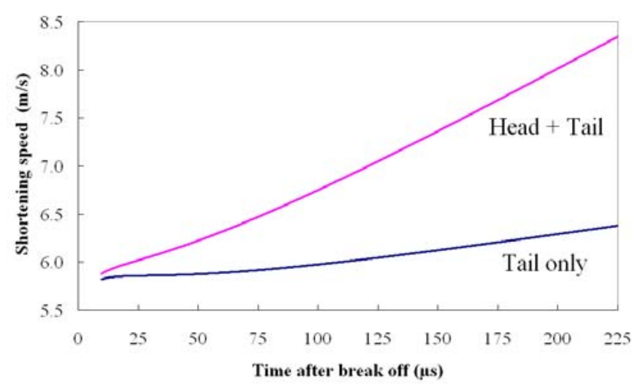

Figure 3(c): Shortening speeds predicted for long ligaments (initial length 1 mm), corresponding to the curves and fluid of Figure 3(b) but with a realistic initial axial velocity which varies linearly along the ligament. The 'tail only' curve is almost flat but the overall shortening speed (upper curve) rises to ~ 8.4 m/s at completion of ligament collapse after 225 µs.

Some of the resulting predictions are shown in Figs. 3(a-c), for typical DoD conditions, with a drop speed of 6 m/s and a ligament diameter at break-off of 3.7 µm [2, 8]. As shown in Fig. 3(a) (tail only) the Taylor prediction v_T for the shortening speed of the ligament is fortuitously also ~6.0 m/s.

Discussion of the model predictions

Figs. 3(a-c) show three examples of results for the same, typical DoD printing fluid: (a) is for a cylindrical ligament with finite length, (b) is for a conical ligament of the same mass, and (c) is for the finite conical ligament with an initial linear axial velocity distribution within the ligament. Tail shortening speeds shown by the lower curves in Figs. 3(a-c) were computed from equation (3); the overall head + tail shortening speeds are the upper curves in Figs. 3(a-c), and were determined using equations (3) and (4).

Fig. 3(a) shows the effect of assuming finite ligament length on the overall shortening speed: at short times, the head speed has not had time to slow down as a back reaction to the much lighter tail, but as the tail gains mass it loses acceleration while the head gains it (towards the tail end, from the surface tension which drives the ligament collapse). The speed is ~20% greater than the Taylor prediction, which is shown by the lower curve in Fig. 3(a).

The overall ligament shortening speed in Figure 3(b) is ~ 3% lower than the Taylor prediction for D = 3.7 µm, due to the conical shape. Keller's model [5] for the same value of *b* predicts a tail shortening speed of ~ 0.6 m/s after 225 µs, which is only 1/7 of the value predicted by the present model. Although the volume of the ligament would be only 3% greater, the non-truncated cone assumed by the Keller model would be ~ 50% longer, and would imply an unrealistic ligament break-off point >500 µm behind the nozzle plate. The Keller theory gives unrealistic predictions of shortening speeds, despite the assumption of a conical ligament shape, because real ligaments do not taper down to a point.

Thus after including the effects of finite ligament mass, length, conical shape, and also taking account of the movement of both ends, the overall ligament shortening speed is predicted to be roughly constant and not far from the value predicted from the simplest, Taylor model which ignores all these effects. However, the agreement is fortuitous and results from the cancellation of significant, but opposite, effects which individually change the speed by ~ 30%.

The predictions of jet shortening speed are further altered if the initial axial velocity distribution along the length of the ligament at break-off [8] is also included, as shown by comparing Figs. 3(b) and (c). The latter incorporates an initial constant velocity gradient of ~ 6 m/s per mm. The predicted ligament shortening speed is increased by ~ 2 m/s at 225 µs after break-off, some 30% higher than that predicted by the Taylor model (equation (1). The effect of initial ligament length *L* in equation (3) is significant. The overall collapse time increases with length due to two effects: (i) long ligaments will have a lower velocity gradient (*U/L*) than short ligaments and therefore will have overall shortening speed curves which are less steep than those in Fig. 3(c); and (ii) the overall collapse time ≈ *L*/(shortening speed).

For a typical DoD printing fluid and a drop speed of 6 m/s, the overall ligament shortening speed, attained before completion of the ligament collapse to form the final drop, is thus predicted to be ~30% higher than predicted from the classic result (equation 1).

Comparison with ligament shortening speeds measured for polymer solutions

We have seen that an overall ligament shortening speed of <8.5 m/s is predicted by the proposed model (Fig. 3c), and can now compare this with experimental measurements reported for dilute polymer solutions [2, 7], pure solvents, and other model ink-jet fluids [8]. Fig. 4 shows very rapid shortening, at ~ 18.5 m/s, for polyethylene oxide (molecular weight ~100 kDa) at concentrations of 0.05 and 0.1 wt% in water/glycerol with a viscosity of 11 mPa s. This speed was much higher than that for most other fluids tested, including solvents with similar viscosity and ligament diameter at break-off. Figure 3(c) predicts that the ligament shortening speed should reach only ~ 6.5 m/s after 70 µs. We attribute this higher speed (~ 3 times faster than the model prediction) to the elastic retraction of polymer molecules within the ligament and infer that they must have become extended during ligament formation. For such viscoelastic fluids, it is therefore clear that the effects of elasticity cannot be ignored, and in some cases can even dominate the process of ligament shortening.

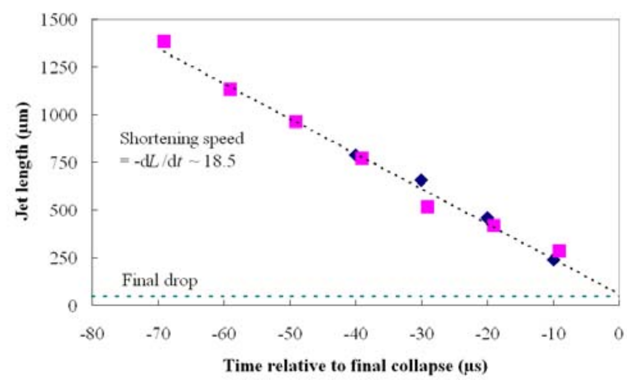

Figure 4: Measured values of jet length for polyethylene oxide (MW ~ 100 kDa) in water/glycerol at viscosity of ~11 mPa s, at (■) 0.1 wt% and (♦) 0.05 wt% concentration, showing jet shortening speeds of ~ 18.5 m/s after break off.

Figure 5(a) shows measured ligament shortening speeds for various concentrations of polymer solutions (polystyrene in diethyl phthalate and polyethylene oxide in water/glycerol) jetted at 6 m/s [2, 8]; there is a marked dependence on the molecular weight for the two polymer systems studied, as well as reasonable agreement with the predictions of the Taylor model (equation 1) and thus also with the present model (equations 3 and 4) for the highest molecular weight polymers. Figure 5(b) shows the same data plotted against the relevant polymer (Zimm) relaxation time [9]. The three fluids with speeds of >15 m/s have similar Zimm times, but quite different molecular weights, and represent both polymer-solvent systems. This behavior of certain polymer solutions may suggest a method of improving the jetting performance of ink-jet fluids that necessarily involve low concentrations of polymer additives. Further details, and further modeling of the polymer physics underlying these marked differences in ligament shortening speed are provided elsewhere [9].

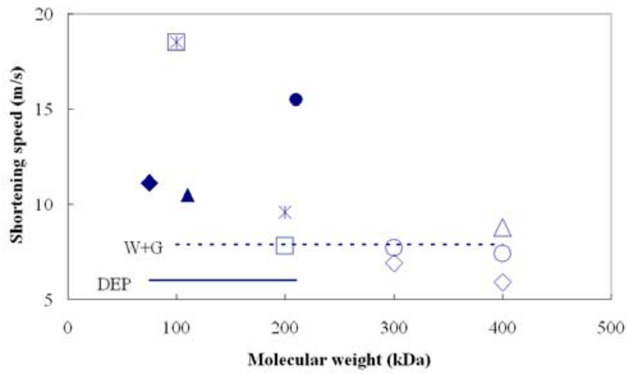

Figure 5(a): Ligament shortening speeds, measured for various polymer solutions jetted at 6 m/s [2, 7, 8]. The solid symbols are for PS+DEP and the open symbols for PEO in water/glycerol, at different molecular weights and concentrations. The horizontal lines show the predictions from equation (1) for the two pure solvents (DEP = diethyl phthalate and W+G = water/glycerol).

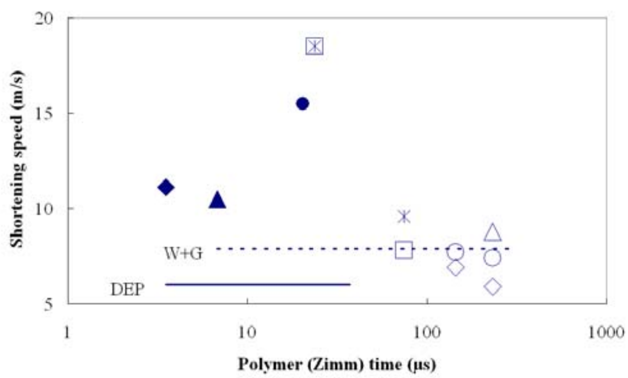

Figure 5(b): Data from Figure 5(a) plotted against the Zimm relaxation time for the polymer-solvent systems [9]. There are 3 data points above 15 m/s, two of them happen to coincide as they did in Figure 5(a), corresponding to Figure 4 for PEO in water/glycerol, the other corresponds to PS(210 kDa) in DEP.

Conclusions

An improved model for jet ligament shortening process has been compared with the classical theory (originally due to Taylor, for cylinders, and extended by Keller to other geometries) that ignores the true ligament shape (a truncated cone) and also ignores the fact that the ligament contains an axial velocity gradient. Predictions from the current model fortuitously lie close to those from the Taylor model, and both models fail to predict the much higher shortening speeds observed for some dilute polymer solutions. The results suggest that the effects of polymer elasticity cannot be ignored, and in some cases may even dominate the process of ligament shortening.

Acknowledgements

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Author Biography

Stephen Hoath received his BA in physics (1972) and his DPhil in nuclear physics (1977) from the University of Oxford. Since then he has worked in UK academia and high tech industry, joining the University of Cambridge Ink-jet Research Centre in 2005. His work there focused on experiments and analysis of dilute polymer fluid Drop on Demand ink-jets. He is a chartered scientist, engineer, physicist, IOP and IS&T member.